Lec 16: 10/21/2013

Epoch of Recombination:

As we saw, neutrinos decouple from the primordial plasma at $t \sim 1$ sec. Henceforth, we have a coupled plasma of photons, electrons and baryons (mainly protons and α particles). The photon scattering off the free electrons results in a pressure that keep the components of the plasma coupled together. The efficient photon-electron scattering also implies that the plasma is opaque to photons. As the temperature decreases, due to expansion, electrons start to get bound to protons and α particles to form neutral atoms. Eventually, (almost) all of the electrons are bound in neutral atoms. As a result, photons can move freely and the universe becomes transparent. This is the epoch of recombination a snapshot of it is provided by the CMB photons.

Let us start with a quick account of recombination and the important physical processes that are involved.

Recombination (Preview):

The important processes are:

(1) Electron-photon scattering:

$e^- + \gamma \rightarrow e^- + \gamma$

At temperatures $\ll m_e$, the scattering can be considered as Thomson scattering, which has the following rate:

$\Gamma_T = n_e \sigma_T$ $\qquad \sigma_T = \frac{2}{3} \times 10^{-24}\ cm^2 = \frac{2}{3}$ barn (1 barn $\equiv 10^{-24}\ cm^2$)

Here n_e is the number density of free electrons, and σ_T is the cross section for Thomson scattering. σ_T can be calculated from classical electrodynamics.

(2) Electron-proton combination and Hydrogen ionization:

$H + \gamma \rightleftarrows p + e^-$

In thermal equilibrium, at $T \ll m_e$, we have:

$$n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \exp\left[-\left(\frac{m_p - \nu_p}{T}\right)\right]$$

$$n_e = 2\left(\frac{m_e T}{2\pi}\right)^{3/2} \exp\left[-\left(\frac{m_e - \nu_e}{T}\right)\right] \qquad (I)$$

$$n_H = 4\left(\frac{m_H T}{2\pi}\right)^{3/2} \exp\left[-\left(\frac{m_H - \nu_H}{T}\right)\right]$$

$$n_\gamma = 2\frac{\zeta(3)}{\pi^2} T^3$$

Here 2, 2, 4 are the spin degrees of freedom for p, e, H respectively. Also, ν_p, ν_e, ν_H denote the chemical potentials of p, e, H respectively. These non-zero chemical potentials indicate a conserved number associated with the corresponding particle. In the case of e and p this conserved number is the electric charge. In the case of H baryon number is the conserved number. We note that $\nu_\gamma = 0$ since there are no conserved numbers (like the electric charge or the baryon number) for the photons. This can be seen from reactions that do not preserve the number of photons, for example:

$$e^- + \gamma \to e^- + \gamma + \gamma$$

Ignoring the ^4He and α particles in the plasma (note that there is one ^4He for every 12 H, and one α particle for every 12 p), we have:

$$n_B = n_p + n_H \qquad (\mathrm{II})$$

Here n_B is the number density of baryons. Also, charge neutrality of the plasma implies that:

$$n_p = n_e \qquad (\mathrm{III})$$

In thermal equilibrium the sum of chemical potentials on the two sides of a reaction are equal, which leads to:

$$\nu_e + \nu_p = \nu_H \qquad (\mathrm{IV})$$

From Eqs. (I, IV), after using $m_H \simeq m_p$, we find:

$$n_H = n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} \exp\left(\frac{B}{T}\right) \qquad (B \equiv m_p + m_e - m_H = 13.6\ \mathrm{eV})$$

The fractional ionization X_e is defined as:

$$X_e = \frac{n_p}{n_B}$$

After using Eqs. (II, III) and the above expression for n_H, we find:

$$\frac{1-X_e^{eq}}{(X_e^{eq})^2} = \frac{4\sqrt{2}\,\zeta(3)}{\sqrt{\pi}}\,\eta\left(\frac{T}{m_e}\right)^{3/2} \exp\left(\frac{B}{T}\right) \qquad (\mathrm{V})$$

Here $\eta \equiv \frac{n_B}{n_\gamma} \simeq 6\times10^{-10}$, and the superscript "eq" denotes the value of X_e in thermal equilibrium.

Eq. (V) is called the "Saha ionization equation". It can be used to find the equilibrium value of the fractional ionization at a given temperature T. It is seen from the Saha equation that $X_e^{eq} \to 0$ as $T \to 0$. However, in an expanding universe the decrease in temperature is caused by the expansion. Like any other process, $p + e^- \to H + \gamma$ drops out of equilibrium when X_e is very small as p and e^- can no longer find each other and combine efficiently. This implies that X_e will not follow the equilibrium value X_e^{eq} below some temperature, and hence there will be a residual ionization fraction X_∞

as $T \to 0$. As we will find later, this residual value is:

$$X_\infty \simeq 3 \times 10^{-3}$$

Therefore, about 0.3% of electrons stay free and unbound at late times.

We can define recombination time as a time when X_e^{eq} is small enough. For example, taking this to be $X_e^{eq} \sim 10\%$, we find the corresponding temperature T_{rec} from the Saha equation:

$$T_{rec} \sim 0.3\ eV$$

Considering that the current temperature of relic photons is $T_0 \simeq 2.72\,°K$, we can find the redshift at the time of recombination:

$$1 + z_{rec} = \frac{T_{rec}}{T_0} \Rightarrow z_{rec} \sim 1200 - 1400 \quad (\text{VI})$$